

# Poster Abstract: On the Scaling Laws of Dense Wireless Sensor Networks

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## ABSTRACT

We characterize the scaling laws of many-to-one dense wireless sensor networks and derive conditions governing the observability of random fields by such networks. We further extend our results to the multimodal case where the sensors observe multiple random processes simultaneously. Quite interestingly, our results show that an *unbounded* number of spatially bandlimited Gaussian processes can be observed simultaneously by a dense multimodal wireless sensor network.

## Categories and Subject Descriptors

H.1.1 [Models and Principles]: Systems and Information Theory—*information theory*

## General Terms

Information-theoretic design

## Keywords

Transport capacity, observability, source/channel coding separation, sensor cooperation

## 1. INTRODUCTION

We consider dense wireless sensor networks deployed to observe arbitrary random fields. The requirement is to reconstruct an estimate of the random field at a certain *collector node*. This creates a *many-to-one* data gathering wireless channel. We show that the transport capacity of such a network scales as  $\Theta(\log(N))$  when the number of sensors  $N$  grows to infinity and the total average power remains fixed. We use this result to derive sufficient and necessary conditions that characterize the set of *observable* random fields by dense sensor networks. We then extend our results to the *multimodal* case where the sensors observe multiple random processes, and the requirement is to reconstruct an estimate of each random process at the corresponding collector node. We show that it is possible to observe an unbounded number of processes ( $O(N^\beta)$ ) such that the transport capacity scales as  $\Theta(\log(N))$  for each of the observed processes, when the number of sensors  $N$  grows to infinity and the total

average power remains fixed. We then use our observability conditions to show that it is possible to simultaneously observe an *unbounded* number of continuous, *spatially bandlimited* Gaussian processes, using a source/channel coding separation scheme.

## 2. THE MANY-TO-ONE CHANNEL

We consider the  $N$  sensor nodes to be distributed uniformly over the perimeter of a circle with a unit radius, while the collector node is at the center. Our results hold for arbitrary single dimensional networks and also extend to the planar scenario. Moreover for large  $N$ , the results hold with high probability for a uniformly random distribution of sensors. We assume that the sensor nodes are equipped with receivers. Our result holds whether the sensor nodes can transmit and receive simultaneously or can do only one task at a time. We denote the random variable observed by node  $j$  at time  $k$  as  $u_j[k]$ . We assume that both the transmitting and receiving nodes have perfect knowledge of the channel between them. We also assume that the joint distribution of all the observations is known *a-priori* at all sensor nodes, which facilitates Slepian-Wolf distributed source coding. We substitute the maximum bit rate constraint of [2] with a constraint on the total average power consumed by the network. Together with the finite bandwidth of the shared wireless medium, we believe that this is a more faithful representation of the wireless channel constraints. We define the *transport capacity* of the many-to-one channel  $C_N$  as the maximum number of bits that can be transported from the  $N$  sensor nodes to the collector per unit time.

**THEOREM 1.** *The transport capacity of the many-to-one channel is  $C_N = \Theta(\log(N))$ .*

To show the achievability of the result, we propose a simple transmission protocol that exploits the high density of the nodes to facilitate cooperative transmission. The main idea is to allow every node to distribute its information to its neighbors, who can then cooperate to transmit the information to the collector. Our transmission protocol does not require the use of any sophisticated multi-user detection schemes. One of the significant implications of Theorem 1 is that one can achieve an *unbounded* transport capacity for the many-to-one channel as the number of nodes grows to infinity with only *finite* total average power. Quite interestingly, the same result holds even if the total power is allowed to scale linearly with  $N$ . One can attribute this limitation

to the bottleneck resulting from the need to deliver all the information to a single collector node. It is also interesting to note that spatial reuse does not factor prominently in the many-to-one scenario.

We develop necessary and sufficient conditions on the observability of random fields by dense wireless sensor networks using the transport capacity result in Theorem 1. Our definition of observability and our results depend largely on whether the observed random variables are discrete or continuous.

*Definition 1.* A discrete random sequence is said to be observable if it can be *detected* at the collector node with arbitrarily small probability of error for a certain allocation of a finite bandwidth and a finite total average power to the network.

For discrete sources, we get the following condition for observability.

**THEOREM 2.** *A spatial random sequence is observable if and only if  $H(u_1[k], u_2[k], \dots, u_N[k]) \leq \Theta(\log(N))$ , where  $H(\cdot, \cdot, \cdot)$  refers to the joint entropy. Moreover, this result holds even if we restrict ourselves to schemes where source and channel coding are performed separately (i.e., source/channel coding separation).*

This separation result establishes the achievability of the same scaling law as the optimal scheme. For continuous sources, our results are limited to single dimensional sensor networks observing spatially stationary random processes.

*Definition 2.* A continuous random process is said to be observable if it can be *estimated* at the collector node with a finite non-zero mean square error for a certain allocation of a finite bandwidth and a finite total average power to the network.

We use the simple approach for distributed lossy source coding proposed in [2] to arrive at the following theorem. This approach is composed of single dimensional quantization followed by distributed Slepian-Wolf coding for the resulting discrete-valued random sequence.

**THEOREM 3.** *All Gaussian spatially bandlimited processes are observable by dense wireless sensor networks as  $N \rightarrow \infty$ . Moreover, this result holds even if we restrict ourselves to the scheme composed of a lossy source encoder concatenated with the proposed cooperative transmission strategy (i.e., source/channel coding separation).*

While Theorem 3 is stated for bandlimited processes, the result extends to other processes that generate  $\leq \Theta(\log(N))$  bits per transmission symbol. The Gaussian process with  $R(d) = e^{-d^2}$  given in [2] is one example of such processes.

### 3. AN EXTENSION TO THE MULTIMODAL SENSOR NETWORK

We extend our results to the scenario where the sensors observe multiple random processes simultaneously. The sensors are required to transmit the information observed about each process to a corresponding collector node. We define the *transport capacity* w.r.t. the  $k^{th}$  process  $C_N(k)$  as the maximum number of bits, belonging to the  $k^{th}$  process, that can be transported from the  $N$  sensor nodes to the  $k^{th}$  collector node per unit time.

**THEOREM 4.** *The transport capacity of the multimodal network w.r.t. each of the observed processes, is  $\Theta(\log(N))$ .*

To show the achievability of the result, we extend the cooperative transmission protocol, used in the many-to-one case, to the multimodal case. While the contention for the single destination node inhibits spatial reuse in the many-to-one case, spatial reuse factors prominently in the multimodal case due to the availability of multiple collector nodes.

**THEOREM 5.** *The multimodal network is capable of observing  $M = N^\beta$  identical processes simultaneously, such that the transport capacity w.r.t. each process is  $\alpha \log(N)$ , where  $\beta \leq \frac{2\delta}{4\delta+1} - \frac{\alpha(2\delta+1)}{4\delta+1}$  and  $\delta$  is the path loss exponent.*

One of the significant implications of Theorems 4 and 5 is that one can observe an *unbounded* number of processes simultaneously and achieve an *unbounded* transport capacity for each of the processes using only *finite* total average power, as the number of nodes grows to infinity. These results can be combined with the conditions on the observability of random processes by dense wireless sensor networks [1] to yield the following theorem.

**THEOREM 6.** *The multimodal network is capable of observing  $M = N^\beta$  identical Gaussian spatially bandlimited processes as  $N \rightarrow \infty$ , where  $\beta \leq \frac{2\delta}{4\delta+1} - \frac{\alpha(2\delta+1)}{4\delta+1}$  and  $\alpha$  depends on the spatial bandwidth of the processes. Moreover, this result holds even if we restrict ourselves to a scheme comprising of a lossy source encoder (single dimensional quantization followed by Slepian-Wolf distributed coding) concatenated with the proposed cooperative transmission strategy (i.e., source/channel coding separation).*

While Theorem 6 is stated for bandlimited processes, the result extends to other processes that generate  $\leq \Theta(\log(N))$  bits per transmission symbol. The separation result establishes the achievability of the same scaling law as the optimal scheme, for the per-process transport capacity.

### 4. REFERENCES

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