Wireless Sensor Networks with Rayleigh Fading Channels

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The Rayleigh fading channel model

- characterizes the wireless channel more accurately than the “disk model”.
- permits a separate analysis of noise and interference issues.
- shows that the benefit of short-hop routing is smaller than often assumed.
- reveals the benefit of time and path diversity schemes.

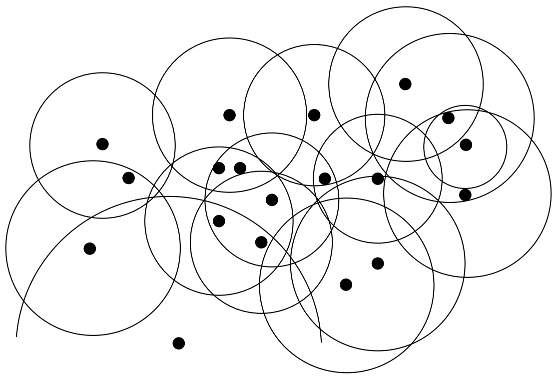
1 – The Geometric "Disk Model"

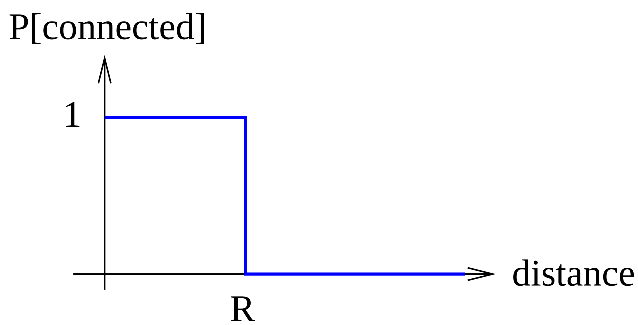
Two versions, the "protocol model" and the "physical model".

In these models, two nodes receive each other's packets with 100% reliability if

- "protocol model": their (Euclidean) distance d is smaller than the so-called transmission radius R, and if no other node in within the receiver's radius is transmitting at the same time.
- "physical model": the SINR is bigger than some threshold.

The protocol model wrongly suggests that scaling every node's transmit power leads to an increasing number of collisions.





Connectivity function

Both versions do not take into account fading, *i.e.*, temporal and spatial variations in the path loss due to multipath propagation, obstacles, and mobility. With fading, the SINR is a random variable, not a deterministic quantity.

2 – The Rayleigh Fading Link Model

- Narrowband block Rayleigh fading with additive noise of variance N_0.
- The received power Q is exponentially distributed with mean $\bar{Q}$. Over a transmission of distance $d = \|x_i - x_j\|_2$ with an attenuation d^α, we have $\bar{Q} = P_0 d^{-\alpha}$.
- I denotes the interference, *i.e.*, the received power from all undesired transmitters. (Interferers are also subject to Rayleigh fading.)
- The *reception probability* p_r over a link is a function of the SINR γ:
  The reception is successful if

$$\gamma := \frac{Q}{N_0 + I} \geqslant \Theta \,.$$

- So, we have

$$p_r = \mathbb{P}[Q \geqslant \Theta(I + N_0)] \,.$$

The Rayleigh Fading Link Model

Theorem: The reception probability $\mathbb{P}[\gamma \geqslant \Theta]$ can be factorized into the reception probability of a zero-noise network and the reception probability of a zero-interference network.

Proof: Let Q_0 be the received power from the desired transmitter and Q_i, $i = 1, \ldots, k$, the received power from k interferers.

$$\begin{aligned} p_r &= \mathbb{P}[Q_0 \geqslant \Theta(I + N_0)] \\ &= \exp\Big(-\frac{\Theta(I + N_0)}{\bar{Q}_0}\Big) \\ &= \int_0^\infty \cdots \int_0^\infty \exp\Big(-\frac{\Theta(\sum_{i=1}^k q_i + N_0)}{\bar{Q}_0}\Big) \prod_{i=1}^k p_{Q_i}(q_i)\, \mathrm{d}q_1 \cdots \mathrm{d}q_k \\ &= \underbrace{\exp\Big(-\frac{\Theta N_0}{P_0 d_0^{-\alpha}}\Big)}_{p_r^N} \cdot \underbrace{\prod_{i=1}^k \frac{1}{1 + \Theta \frac{P_i}{P_0} \left(\frac{d_0}{d_i}\right)^\alpha}}_{p_r^I} . \end{aligned}$$

This allows an independent analysis of noise (part 3) and interference (part 4).

3 – Impact on Routing [1]

Multihop connections:

The end-to-end reception probability of a chain of n nodes is

$$p_{EE} = \prod_{i=1}^{n} e^{-\Theta/\bar{\gamma}_i} = e^{-\Theta \sum_{i=1}^{n} \frac{1}{\bar{\gamma}_i}}$$

where $\bar{\gamma}_i$ denotes the mean SNR at link i.

Energy to cover a distance nd with reliability p_{D}:

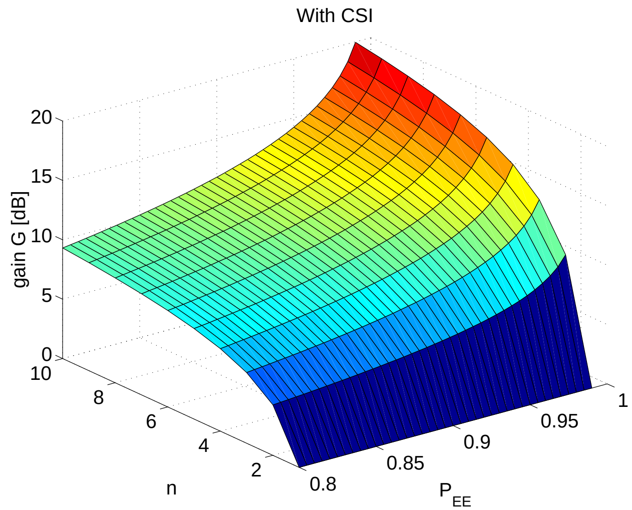
- Single hop: $E_1 = n^{\alpha} E_0$.
- n hops: A reception probability $p_r = \sqrt[n]{p_{\mathrm{D}}}$ is required at each hop. Since $\ln p_{\mathrm{D}} = n \ln p_r$, the total energy in this case is $E_n = n \cdot n\, E_0$.

where E_0 is the energy required for a transmission over distance d with probability p_{D}, *i.e.*, $E_0 := -d^{\alpha}\Theta N_0 / \ln p_{\mathrm{D}}$.

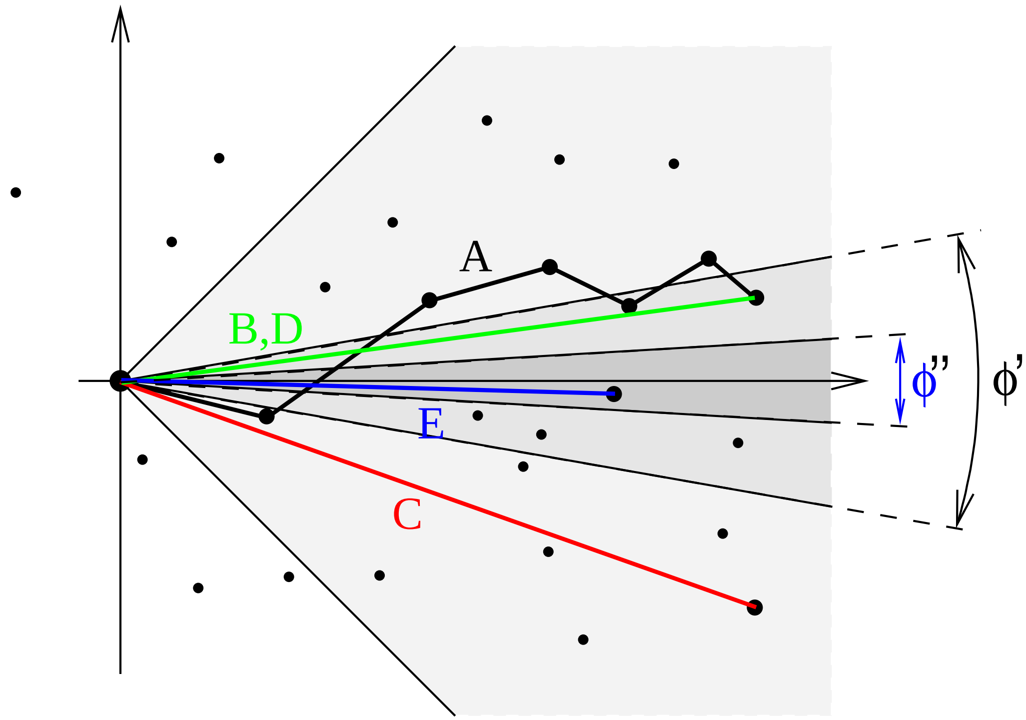
So, for $\alpha = 2$, there is no benefit at all in multihop routing. For higher α, the benefit is smaller than often assumed, especially when path efficiencies and delays are taken into account.

Impact on Routing

Comparison of different routing strategies:
The path efficiency increases with the length of the hops. Permitting equal delay for long-hop routing, retransmissions can be used to increase the efficiency.



Energy gain of long-hop routing with CSI ($\alpha = 3$).



A: nearest-neighbor (short-hop) routing over n hops.
B: long-hop routing directly to n-th node.
C: long-hop routing to n'-th neighbor in sector ϕ.
D: long-hop routing to n-th neighbor in sector $\phi' < \phi$.
E: long-hop routing to nearest neighbor in $\phi'' < \phi'$.

4 – Throughput Limit in Heavy Traffic [2]

Throughput:
Define the long-term average throughput as

$$g = \lim_{j\to\infty} \frac{1}{j} \sum_{k=1}^{j} \mathbb{P}[\text{transmit}_k]\, p_r[k]\,,$$

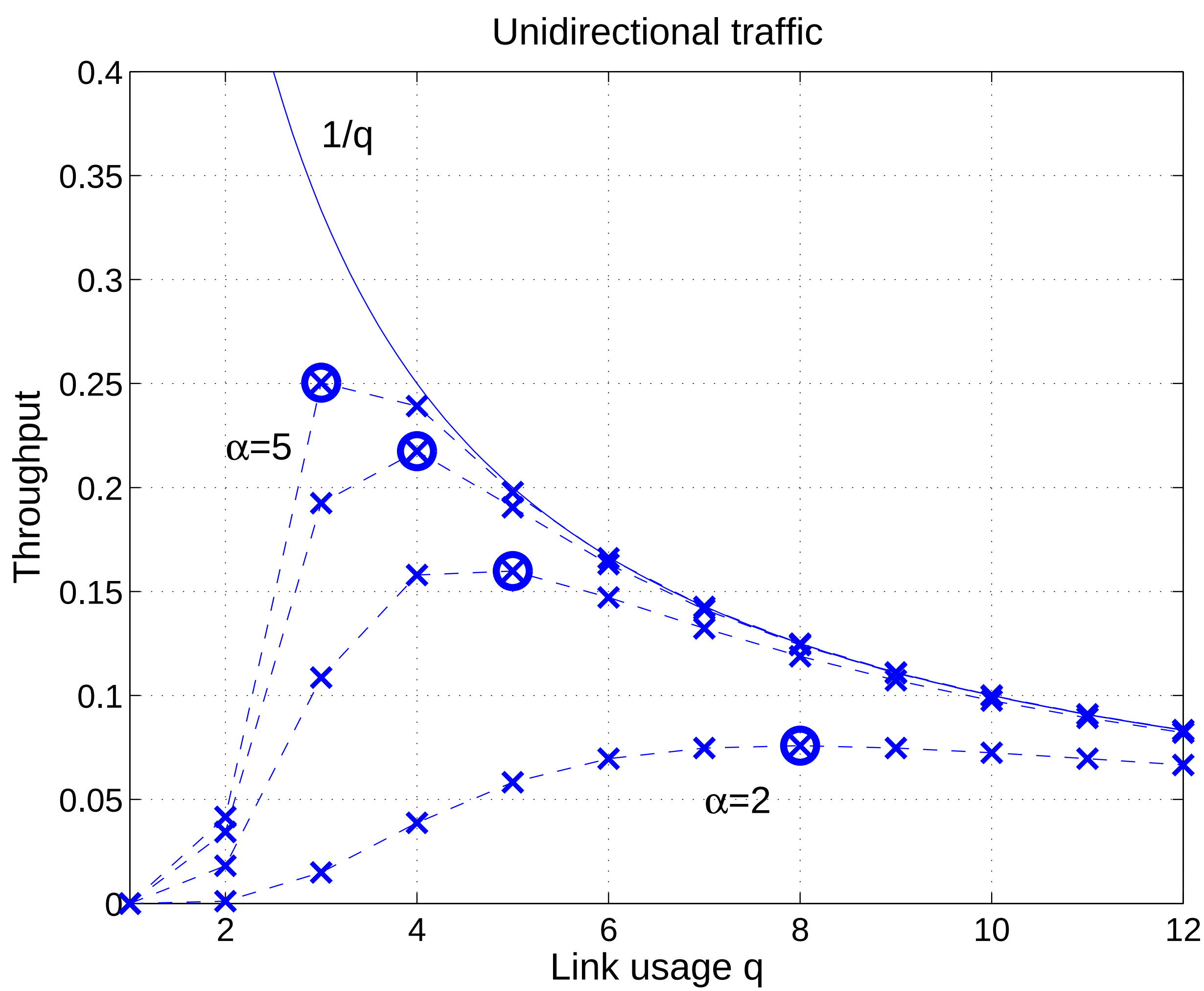
where $p_r[k]$ denotes the reception probability of a given node in timeslot k and $\mathbb{P}[\text{transmit}_k]$ is the probability that the node actually transmits in timeslot k.

Long equidistant line networks:
Optimum throughput if every node transmits to its right neighbor:

$$g_{\max} = \max_{q\in\mathbb{N}} \left(q \prod_{i=1}^{\infty} \underbrace{\left(1+\Theta(qi-1)^{-\alpha}\right)}_{\text{right neighbors}} \underbrace{\left(1+\Theta(qi+1)^{-\alpha}\right)}_{\text{left neighbors}} \right)^{-1}$$

q is the link utilization (every q-th link is used at the same time).

Throughput Limit in Heavy Traffic

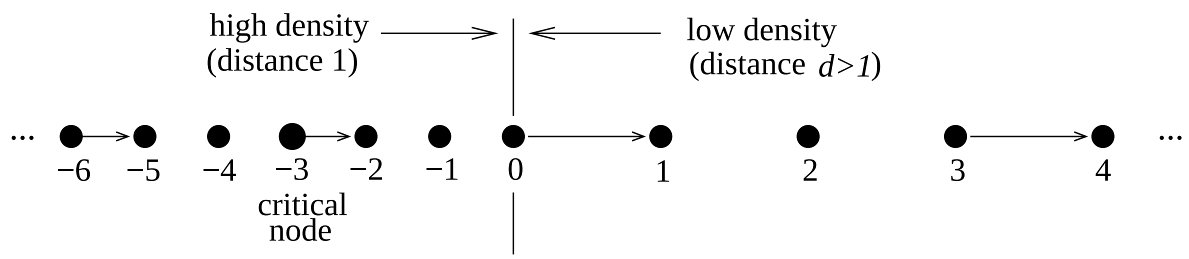


Received packets per timeslot and node for $\Theta = 10$ and $\alpha = 2, 3, 4, 5$.

Throughput Limit in Heavy Traffic

Line network with abruptly changing node density:

The node in the high-density part that transmits at the same time as the boundary node is the *critical* node.



Taking only the boundary node into account as an interferer, we get:

$$p_r \approx \frac{1}{1+\Theta\left(\frac{d}{q-1}\right)^{\alpha}} \qquad \text{and} \qquad q_{\text{opt}} \approx d \cdot \left(\frac{\Theta\, p_{\text{opt}}}{1-p_{\text{opt}}}\right)^{\frac{1}{\alpha}} + 1\,. \tag{1}$$

So, $q_{\text{opt}} - 1 \approx c \cdot d$ with a coefficient c that depends on Θ, p_{opt}, and α. Since $1/q_{\text{opt}}$ is an upper bound on the throughput, we have $g_{\max} \lessapprox 1/(cd+1)$.

$\Longrightarrow$ The maximum achievable throughput over a junction with node density ratio d is approximately inversely proportional to d.

Throughput Limit in Heavy Traffic

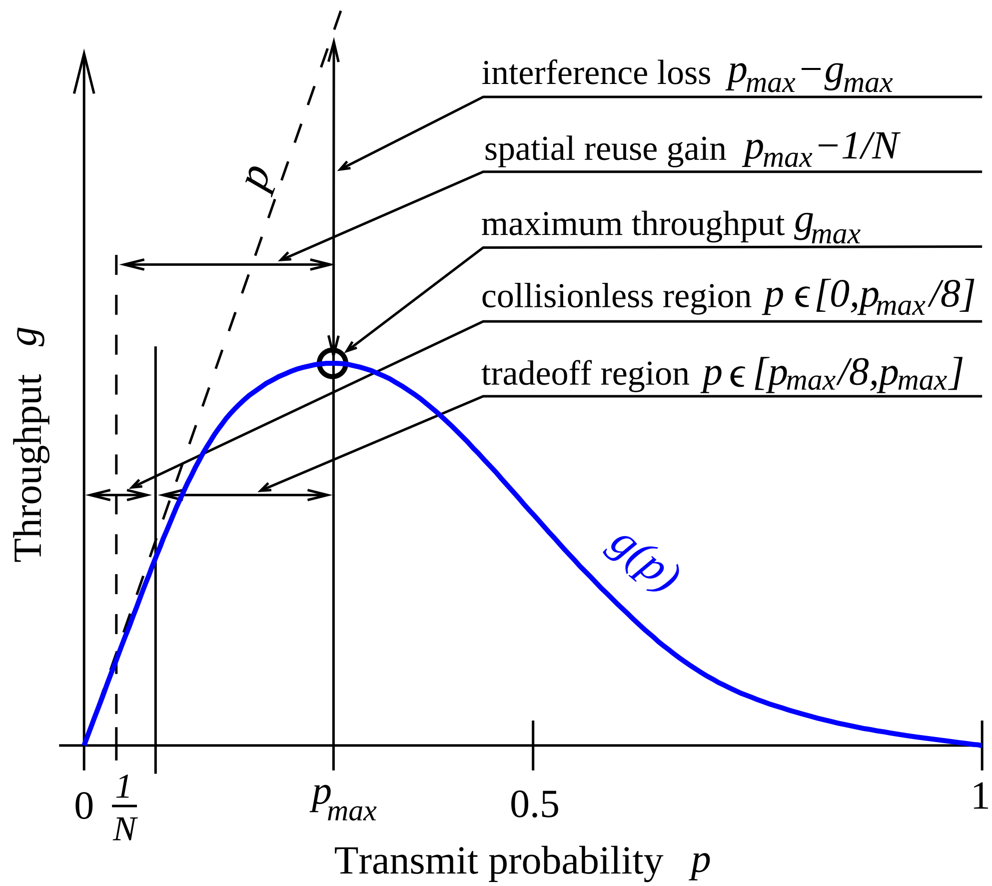
To derive a lower throughput limit, we consider a simple random MAC.

All nodes are transmitting packets independently in every timeslot with the same transmit probability p.

The (per-node) throughput in a network of N nodes is a polynomial in p of order N:

$$g(p) = \sum_{k=1}^{N-1} c_k p^k (1-p)^{N-k}$$

There is an optimum $g_{\max}(p_{\max})$ with $0 < p_{\max} < \frac{1}{2}$.



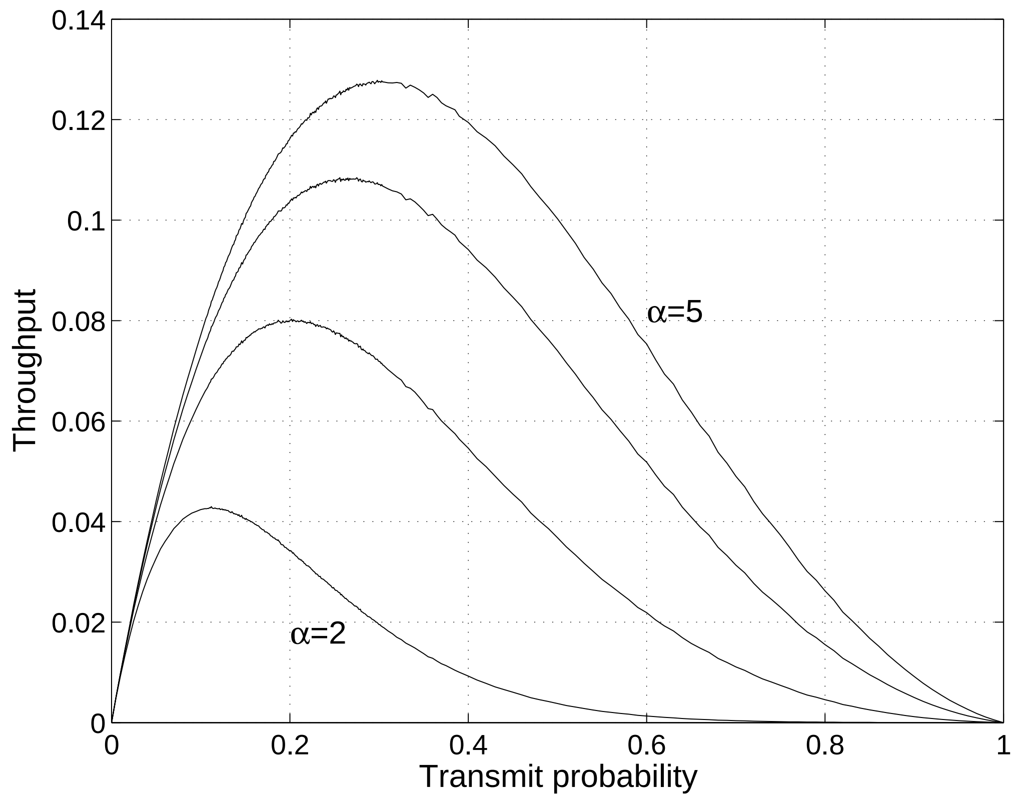
Generic throughput polynomial. “Collisionless” means less than 10% packet loss.

Throughput Limit in Heavy Traffic

Equidistant line network with random MAC:

The packet loss probabilities at $p_{\max}$ are substantial: 60%$\pm$2%, independent of α.
- at $p_{\max}/2$: 37%$\pm$2%
- at $p_{\max}/4$: 20%$\pm$1%.
- at $p_{\max}/8$: 10%.

These results only take interference into account, not noise. They are accurate for large transmit powers, where $p_r^N \approx 1$.

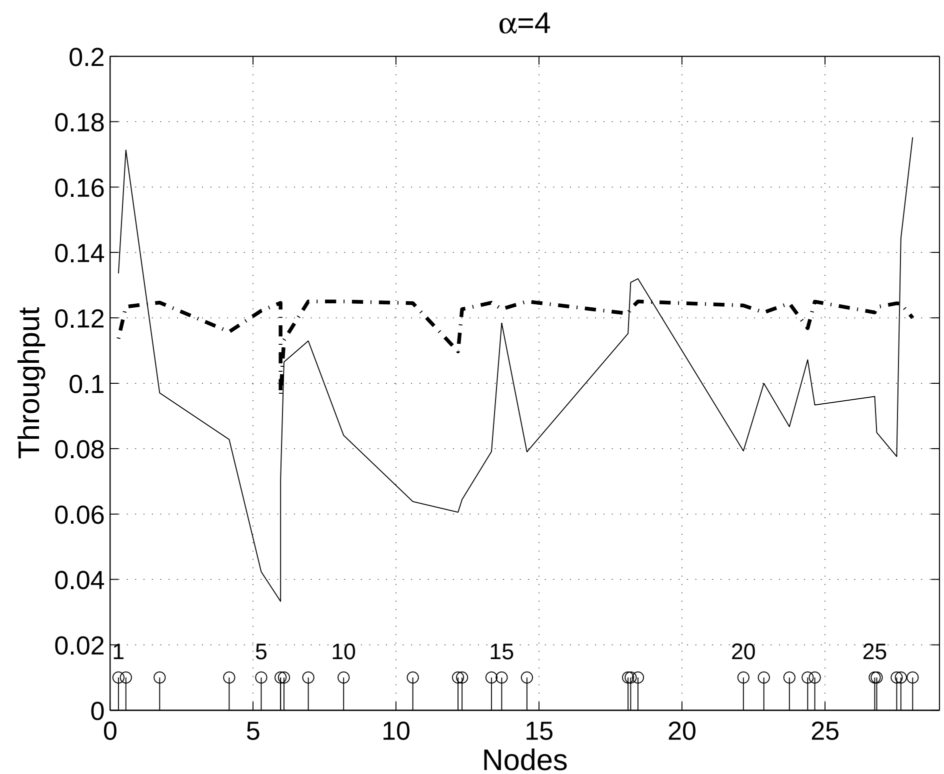


Throughput Limit in Heavy Traffic

Line network with uniformly random node distribution:

- Internode distances are exponentially distributed.
- Throughput varies from node to node.
- Power control is needed to avoid links with very low p_r^N.

On average, the end-to-end throughput is about 50% of the throughput of the equidistant network, at much higher energy consumption.



Per-node throughput for random MAC (solid line) and optimum MAC (dash-dotted line) for a typical 30-node line network with $\alpha = 4$.

5 – Diversity Schemes and Energy and Delay Balancing [3–5]

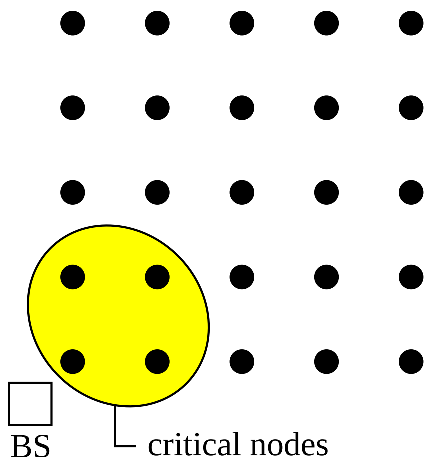
Let $R := \Theta/\bar{\gamma}$ be the normalized NSR. For a single link, we have $p_r = e^{-R}$. For $p_r \lessapprox 1$ $(R \ll 1)$, R is the packet loss probability, *i.e.*, $R \approx 1 - p_r$.

Since, for $(r_1, r_2, \ldots, r_n) \in (\mathbb{R}_0^+)^n$,

$$1 - \prod_{i=1}^{n} (1 - e^{-r_i}) \geqslant e^{-\prod_{i=1}^{n} r_i} \,,$$

packet loss probabilities multiply when a packet is retransmitted. This permits a convenient analysis and design of path and time diversity schemes [3].

The Rayleigh fading model also proves useful to achieve energy balancing [4] and delay balancing [5] in typical sensor network traffic scenarios, where all data have to be relayed to a single base station. typical sensor network traffic.



6 – Concluding Remarks

- Whereas fading channels have been studied in detail at the physical layer, their impact on higher layers is largely an open problem.
- The Rayleigh fading model incorporates fading in a convenient way, *i.e.*, it is analytically tractable and provides isolation between noise and interference.
- With fading, the energy benefit of short-hop routing becomes much less significant. If, in addition, the end-to-end delay is considered, minimum-hop routing may outperform maximum-hop (or shortest-hop) routing.
- Diversity schemes can be assessed, and power management and scheduling strategies can be developed for fading channels.

Acknowledgment:
The partial support of the NSF (ECS02-25265) and the DARPA/IXO-NEST Program (AF-F30602-01-2-0526) is gratefully acknowledged.

References

[1] M. Haenggi, "On Routing in Random Rayleigh Fading Networks," *IEEE Transactions on Wireless Communications*, 2003. Submitted for publication. Available at `http://www.nd.edu/~mhaenggi/routing.pdf`.

[2] M. Haenggi and X. Liu, "Fundamental Throughput Limits in Rayleigh Fading Sensor Networks," *IEEE Journal on Selected Areas in Communications*. Submitted for publication. Available at `http://www.nd.edu/~mhaenggi/jsac03.pdf`.

[3] M. Haenggi, "A Formalism for the Analysis and Design of Time and Path Diversity Schemes in Wireless Sensor Networks," in *The 2nd International Workshop on Information Processing in Sensor Networks (IPSN'03)*, (Palo Alto, CA), pp. 417–431, Apr. 2003. Available at `http://www.nd.edu/~mhaenggi/ipsn03.pdf`.

[4] M. Haenggi, "Energy-Balancing Strategies for Wireless Sensor Networks," in *IEEE International Symposium on Circuits and Systems (ISCAS'03)*, (Bangkok, Thailand), May 2003. Available at `http://www.nd.edu/~mhaenggi/iscas03.pdf`.

[5] M. Xie and M. Haenggi, "Performance Analysis of a Priority Queueing System Over Rayleigh Fading Channels," in *41st Annual Allerton Conference on Communication, Control, and Computing*, (Monticello, IL), Oct. 2003.